experimental data

1. When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds). This graph is shown in fig. 1.
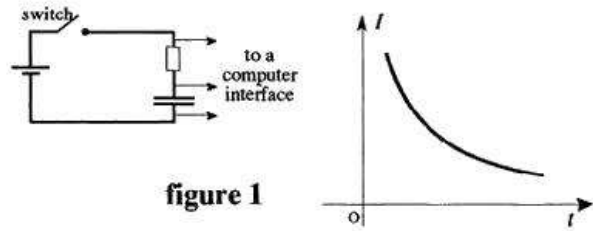
figure 1

In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.
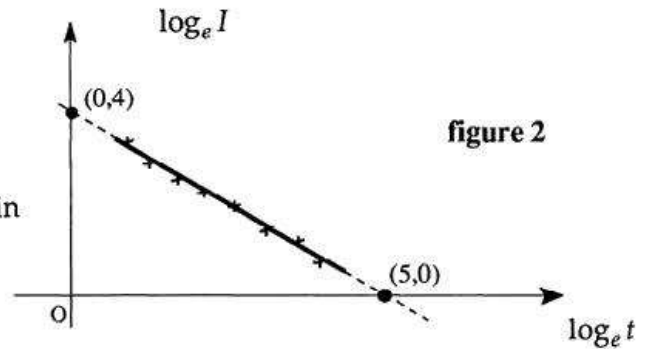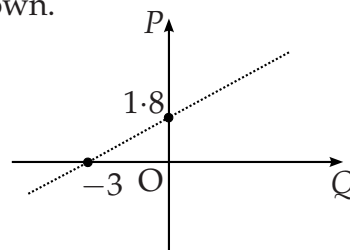
$\log_e I$

(0,4)

figure 2

(5,0)

$\log_e t$

(a) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$. **(3)**

(b) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. **(4)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	3	C	CR	G2, G3		1993 P2 Q10
(b)	4	A/B	CR	A33		

(a) •¹ $m = -\frac{4}{5}$ stated or implied

•² $y = mx + 4$ stated or implied

•³ $\log_e I = -\frac{4}{5}\log_e t + 4$

(b) •⁴ $\log_e t^{-\frac{4}{5}}$

•⁵ $\log_e 54.6$

•⁶ $\log_e 54.6t^{-\frac{4}{5}}$

•⁷ $I = 54.6t^{-0.8}$

2. The results of an experiment give rise to the graph shown.

(a) Write down the equation of the line in terms of P and Q. **2**

It is given that $P = \log_e p$ and $Q = \log_e q$.

(b) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	A/B	CR	G3	$P = 0{\cdot}6Q + 1{\cdot}8$	2000 P2 Q11
(b)	4	A/B	CR	A33	$a = 6{\cdot}05,\ b = 0{\cdot}6$	

•^{1} ic: interpret gradient
•^{2} ic: state equ. of line

•^{3} ic: interpret straight line
•^{4} ss: know how to deal with x of $x \log y$
•^{5} ss: know how to express number as log
•^{6} ic: interpret sum of two logs

•^{1} $m = \frac{1{\cdot}8}{3} = 0{\cdot}6$
•^{2} $P = 0{\cdot}6Q + 1{\cdot}8$

Method 1

•^{3} $\log_e p = 0{\cdot}6 \log_e q + 1{\cdot}8$
•^{4} $\log_e q^{0{\cdot}6}$
•^{5} $\log_e 6{\cdot}05$
•^{6} $p = 6{\cdot}05 q^{0{\cdot}6}$

Method 2
$\ln p = \ln aq^b$

•^{3} $\ln p = \ln a + b \ln q$
•^{4} $\ln p = 0{\cdot}6 \ln q + 1{\cdot}8$ *stated or implied by* •^{5} *or* •^{6}
•^{5} $\ln a = 1{\cdot}8$
•^{6} $a = 6{\cdot}05,\ b = 0{\cdot}6$

3. The graph illustrates the law $y = kx^n$.

   If the straight line passes through $A(0.5, 0)$ and $B(0, 1)$, find the values of k and n.

   4
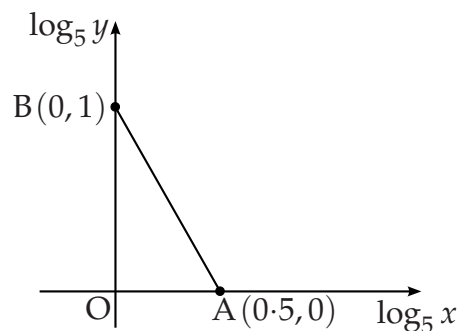
Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	4	A/B	NC	A33	$y = 5x^{-2}$	2002 P1 Q11

$\bullet^1$ ic: interpret graph	$\bullet^1$ $\log_5 y = -2(\log_5 x) + 1$
$\bullet^2$ ss: use log laws	$\bullet^2$ $\log_5 y = \log_5 x^{-2} + \dots$
$\bullet^3$ ss: use log laws	$\bullet^3$ $\dots + \log_5 5$
$\bullet^4$ pd: solve log equation	$\bullet^4$ $y = 5x^{-2}$

4. Variables x and y are related by the equation $y = kx^n$.

   The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points $(0, 5)$ and $(4, 7)$, as shown in the diagram.

   Find the values of k and n.
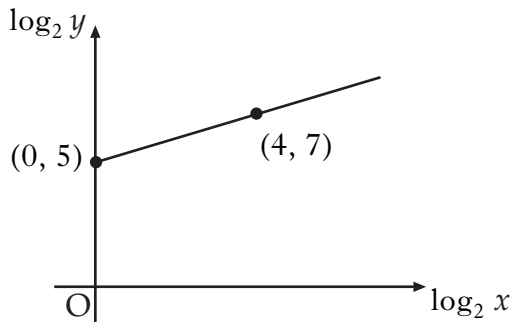
   5

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	5	A	CN	A33	$k = 32, n = \frac{1}{2}$	2011 P2 Q5

$\bullet^1$ ss: introduce logarithms to $y = kx^n$	$\bullet^1$ $\log_2 y = \log_2 kx^n$
$\bullet^2$ ic: use laws of logarithms	$\bullet^2$ $\log_2 y = n \log_2 x + \log_2 k$
$\bullet^3$ ic: interpret intercept	$\bullet^3$ $\log_2 k = 5$ or $\log_2 y = 5$
$\bullet^4$ ic: solve for k	$\bullet^4$ $k = 32$ or 2^5
$\bullet^5$ ic: interpret gradient	$\bullet^5$ $n = \frac{1}{2}$

5. *(a)* The variables x and y arc conncctcd by a relationship of the form $y = ae^{bx}$ where a and b are constants. Show that there is a linear relationship between $\log_e y$ and x. **(3)**

(b) From an experiment some data was obtained. The table shows the data which lies on the line of best fit.

| x	3.1	3.5	4.1	5.2
y	21 876	72 631	439 392	11 913 076

The variables x and y in the above table are connected by a relationship of the form $y = ae^{bx}$. Determine the values of a and b. **(6)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	3	A/B	CR	A33		1998 P2 Q11
(b)	6	A/B	CR	A30		

(a) $\bullet^1$ $\log_e y = \log_e ae^{bx}$

$\bullet^2$ $\log_e y = \log_e a + \log_e e^{bx}$

$\bullet^3$ $\log_e y = \log_e a + bx$

(b) $\bullet^4$ evidence for strategy being carried out

will be appearance of two equations at $\bullet^5$ stage

$\bullet^5$ e.g. $3.1b + \log a = 9.99$, $5.2b + \log a = 16.29$

$\bullet^6$ strategy: know to subtract

$\bullet^7$ $b = 3$

$\bullet^8$ $a = e^{0.69}$

$\bullet^9$ $a = 2$

6. As shown in the diagram, a set of experimental results gives a straight line graph when $\log_{10} y$ is plotted against $\log_{10} y$. The straight line passes through $(0, 1)$ and has a gradient of 2.

Express y in terms of x.

6
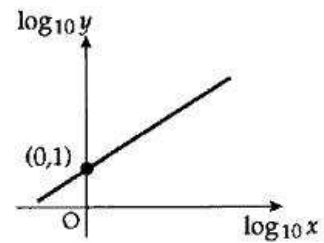
Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	2	C	CN	G3		1990 P1 Q14
	4	A/B	CN	A33, A34		

- •1 use $y = mx + c$
- •2 $\log_{10} y = 2\log_{10} x + 1$
- •3 $\log_{10} y = 2\log_{10} x + \log_{10} 10$
- •4 $\log_{10} y = \log_{10} x^2 + 1$
- •5 $\log_{10} y = \log_{10} 10x^2$
- •6 $y = 10x^2$

7. Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

X $(=\log_e x)$	Y $(=\log_e y)$
2.10	7.00
2.31	7.60
2.40	7.92
2.65	8.70
2.90	9.38
3.10	10.00
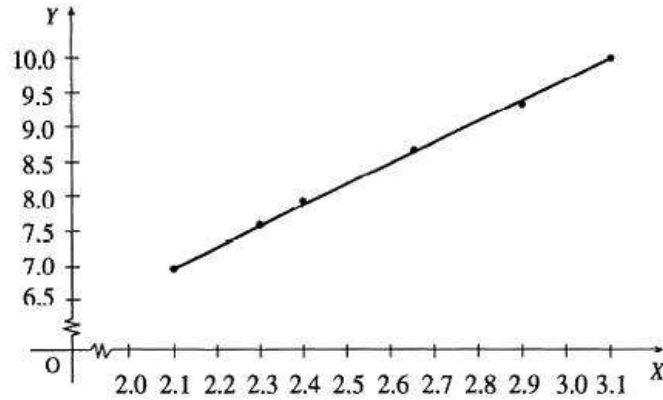
A graph was drawn and is shown above.

(a) Find the equation of the line in the form $Y = mX + c$. **(3)**

(b) Hence find the values of the constants a and b in the relationship $y = ax^b$. **(4)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	3	C	CR	G3		1996 P2 Q9
(b)	4	A/B	CR	A33, A34		

(a) $\bullet^1$ e.g. $m = 3$

     $\bullet^2$ e.g. $8.70 = 3 \times 2.65 + c$ or equiv.

     $\bullet^3$ e.g. $Y = 3X + 0.75$

(b) $\bullet^4$ $\ln y = 3 \ln x + 0 \cdot 7$

     $\bullet^5$ $\ln y = \ln 2.01 x^3$

     $\bullet^6$ $b = 3$

     $\bullet^7$ $a = 2.01$

[END OF QUESTIONS]